**Spiralling Triangles**

<EFOFEX>
id:fxd{c507ee14-daa4-4679-9284-1ab1bff42c24}

FXData:

</EFOFEX>

<EFOFEX>
id:fxd{0aff433f-1cc7-4aa9-b962-77f1a5fc7810}

FXData:
</EFOFEX>We have constructed a spiral shape using a simple process. We start with a right isosceles triangle with short sides 1 unit long.

<EFOFEX>
id:fxd{f7353674-5746-4dea-85c7-b4c9cb92ce3f}

FXData:
</EFOFEX>We then construct a new right triangle on the hypotenuse, keeping the outside edge 1 unit long.

We continue this process to construct the spiral shape at the top of the page.

1. **Calculate the exact length of x, the hypotenuse of the 14th triangle (the blue line on the original diagram).**

[2 Marks]

1. **Describe a pattern that you could use to determine the exact length of the hypotenuses of the triangles in the spiral.**

[2 Marks]

1. **Calculate the length of the hypotenuse of the 104th triangle in the spiral.**

[2 Marks]

<EFOFEX>
id:fxd{f37e5a07-35ab-4ce9-82fd-e394f9c70882}

FXData:

</EFOFEX>

We have continued the pattern up to the 17th triangle and the triangles have begun to overlap. The spiral has completed **one rotation**. A student has hypothesized that, if the pattern was continued further, it would complete a second rotation after another 17 triangles – at triangle 34.

1. **Explain why the student is incorrect.**

[1 Mark]

<EFOFEX>
id:fxd{2222945d-c0b7-4c1d-aa95-f8d1e03df739}

FXData:

</EFOFEX>

1. **Devise a formula that calculates the central angle of a triangle (θ) given the triangle number (n).**

[2 Marks]

1. **Use your formula and a spreadsheet to complete this table. Provide full details of how you determined how many triangles were required to complete the rotations.**

|  |  |
| --- | --- |
| **Number of Rotations** | **Triangle Number** |
| 1 | 17 |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |

[6 Marks]

1. **Calculate the radius of the spiral after five rotations**

[1 Mark]

**Spiralling Triangles - Solutions**

1 and 2.

<EFOFEX>
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FXData:

</EFOFEX>

<EFOFEX>
id:fxe{4f016462-d18a-4a95-b1de-2bb3eb14467e}
FXGP:
FXData:
</EFOFEX>

**Discussion:**

Triangle 1.

<EFOFEX>
id:fxd{8ec13417-1f6c-4c82-999c-a0d588e86b2e}

FXData:

</EFOFEX>

Triangle 2.

<EFOFEX>
id:fxd{bafe5680-e5ca-410e-99ca-269628774c72}
FXGP:
FXData:
</EFOFEX>

Triangle 3.

<EFOFEX>
id:fxd{6015a22d-386c-4bb8-8677-edcdb31613e3}
FXGP:
FXData:
</EFOFEX>

In general

<EFOFEX>
id:fxd{07d3c78f-4076-4e15-813f-a6ac969f3665}
FXGP:
FXData:
</EFOFEX>

3. Using our formula the length of the hypotenuse of the 104th triangle will be <EFOFEX>
id:fxe{598e4787-0fa0-4251-acea-700282003911}
FXGP:
FXData:
</EFOFEX>

4. The student is incorrect because the central angle of each triangle is getting smaller. This means that it will take more than 34 triangles to complete two revolutions. <EFOFEX>
id:fxe{2b45a320-d784-46de-bced-8155db179346}
FXGP:
FXData:
</EFOFEX>

5.

<EFOFEX>
id:fxd{532083c6-d644-4a9a-9bdf-301f57c364f6}
FXGP:
FXData:
</EFOFEX>

Theta is the central angle.

<EFOFEX>
id:fxe{33cdc592-fe78-43ba-b625-3614c3140d28}
FXGP:
FXData:
</EFOFEX>

6.

|  |  |
| --- | --- |
| **Number of Rotations** | **Triangle Number** |
| 1 | 17 |
| 2 | 54 |
| 3 | 110 |
| 4 | 186 |
| 5 | 281 |

<EFOFEX>
id:fxe{39b7aa9b-b079-4191-8cbb-6284ad029dbd}
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Students should provide evidence of using a spreadsheet to complete these values.

<EFOFEX>
id:fxe{775e4358-e383-40ad-b389-f0ec8d06dae0}
FXGP:
FXData:
</EFOFEX>

[Link to sample Excel Spreadsheet.](http://efofex.com/etrp/spiraloftheodorus/dslzgkftvwlulmar/FXTRPSpiralOfTheodorus.xlsx)

7. <EFOFEX>
id:fxe{27b1c7d2-0b51-4410-b4e7-e86cee1544de}
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