**Solving an Optimisation Problem**

This worksheet will show you how to use algebra to solve a problem. There are a number of steps that you need to undertake. You can use this as a template for other questions.

**Step 1 – Read The Problem**

**Find the dimensions of the rectangle with the largest area, that can fit inside the right triangle shown below.**



The first step is to make sure that you understand the problem. In this case the problem is fairly simple – the rectangles need to fit inside the triangle, touching the perpendicular sides, with the corner on the hypotenuse.

We need to find out the sizes of the biggest rectangle.

**Step 2 – Try a Few Examples to Make Sure You Understand**

I will try a few examples, work out the areas and see if I can see any patterns.

From my experiments, I can see that changing the rectangle does change the area and that the maximum area is at least 9.6cm2. **Most importantly, if I change the width, the height is determined by the triangle – I do not get to choose it separately to the width.**

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**Step 3 – Choose ONE Thing to Change, Call it *x***

Often, when you change one thing, all the other things area affected. In my examples, I was changing the width and then measuring the height and calculating the area**. I am going to base everything on the width of the rectangle and call that *x*.** (I don’t have to use *x*, I could use *w* or any other letter. We just traditionally use *x*.)

**Step 4 – Work Out How Changing *x* Changes the Other Things. The Hard Bit!**

It is always very tempting to add another letter (like *y*) and that is OK, providing you eventually get rid of the *y* and only end up with *x’s*. To do that, I need to work out what *y* has to be **in terms of *x***. That means that I have to work out an equation for *y* that only has *x’s* in it.

In this example, I am going to do that using similar triangles.



In the diagram, you can see that the little triangle has sides of (5-*x*) cm and *y* cm and it is also similar to the big triangle which has sides of 8cm and 5 cm. I know that similar triangles have sides in ratio.



**Step 5 – We Have an Equation, Now Graph It!**

Graphs are number pictures and can let you see things that may not otherwise be obvious. Use a graphing calculator or graphing program to graph this function.



From my graph I can see that it appears that the maximum area is 10 square centimetres when the width is 2.5cm. The graph is a parabola and the function is quadratic, so I can use my knowledge of quadratic functions to be sure.

**Step 6 – Use Some Algebra to Solve the Problem**

The line of symmetry of a parabola is calculated using  when the formula is in the form  . I need to rearrange my equation to be  and then the line of symmetry and maximum area can be calculated.



**Step 7 – Answer the Question!**

The last step is so simple that many people forget to do it!

**The largest rectangle that can be fitted into the triangle has a width of 2.5cm and a height of 4cm and has an area of 10cm2**

