**Logo, icon

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**Semicircle Areas**

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**Question**

A series of shapes are constructed from semicircles. The first shape is one large semicircle on the bottom with two smaller semicircles – with radii exactly half of the bottom semicircle – on the top.

<EFOFEX>
id:fxd{8ddb70e2-bb91-405a-bd1e-1c77e60231fc}

FXData:

</EFOFEX>

Each subsequent shape is produced by replacing the rightmost semicircle with two smaller semicircles of half the radius.

<EFOFEX>
id:fxd{61a9a8ba-3b0c-4f8d-84bf-133ae3fa5b26}

FXData:

</EFOFEX>

The next four shapes are shown below.

**<EFOFEX>
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FXData:

</EFOFEX><EFOFEX>
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FXData:

</EFOFEX>**

**<EFOFEX>
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FXData:

</EFOFEX><EFOFEX>
id:fxd{0ff2d63d-d532-45bd-9564-9f910dc53ad3}

FXData:

</EFOFEX>**

Assume that the radius of the first semicircle is 1.

<EFOFEX>
id:fxd{facd7b44-8329-4b52-85ad-1b85a3f14d71}

FXData:

</EFOFEX>

1. **Calculate the area of the first shape shown above.**

As we build further shapes, the area will decrease.

1. **Calculate the areas of the next four shapes in the sequence. You need to show all working.**

You may have noticed that the areas are **almost** based on a geometric sequence.

1. **Use your calculations, and your knowledge of geometric series, to devise a formula that allows you to calculate the area of the nth shape in the sequence.**
2. **Use your formula to calculate the area of the 7th shape in the series. Round your answer to 6 decimal places.**

It is noted that you can continue this process an infinite number of times.

1. **Use your knowledge of infinite geometric series to calculate the area of the shape produced if you continue the process an infinite number of times. Comment on any assumptions you have made.**
2. **Determine if the PERIMETER of the shapes converges or diverges.**

**Solution**

For these solutions we are using the formula <EFOFEX>
id:fxe{92a85491-6492-4cb3-9cac-7ab3e2d8960a}

FXData:

</EFOFEX> for the area of a semicircle.

1. <EFOFEX>
   id:fxe{90b5f840-34c3-42c3-8e8a-e3f711787a1f}

   FXData:

   </EFOFEX>

<EFOFEX>
id:fxd{c1b60c82-eff3-429b-a902-3573f6280dfe}

FXData:

</EFOFEX>

1. <EFOFEX>
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   FXData:

   </EFOFEX>

<EFOFEX>
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FXData:

</EFOFEX>

<EFOFEX>
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FXData:

</EFOFEX>

<EFOFEX>
id:fxe{3d8c1561-45d1-4291-a9c8-f817d4fe3a74}

FXData:

</EFOFEX>

1. The most crucial thing to note is that all but the last term in each sum is a geometric sequence where <EFOFEX>
   id:fxe{f448a7e3-ce04-4c89-bf63-500f49621eb2}

   FXData:

   </EFOFEX>. For example, for the 5th shape in the sequence:

<EFOFEX>
id:fxd{766b90a7-f318-4505-8ae8-b5924517f013}

FXData:

</EFOFEX>

Alternatively, we can take a common factor out before looking at the sum.

<EFOFEX>
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FXData:

</EFOFEX>

Using <EFOFEX>
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FXData:

</EFOFEX>

<EFOFEX>
id:fxd{be7df544-8cb8-48dd-963c-10ffe682efa1}

FXData:

</EFOFEX>

In general:

<EFOFEX>
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FXData:

</EFOFEX>

Note: There are numerous ways of simplifying this equation which could look quite different. Also note that both the numerator and denominator of the formula are divisible by 6 but this is an unlikely simplification for most students.

1. Using our formula

<EFOFEX>
id:fxd{6bb7e93a-2081-463d-b3af-9a694a9252a7}

FXData:

</EFOFEX>

1. Going back to our original construction of the formula, we noted that it was based on the sum of a geometric series.

<EFOFEX>
id:fxd{4cba3b5e-c102-48ed-8376-c143e3b0a262}

FXData:

</EFOFEX>

We can use this information to calculate the area of <EFOFEX>
id:fxe{5bceabba-e927-450e-9cda-5af68cae68f3}

FXData:

</EFOFEX>.

<EFOFEX>
id:fxd{9933d5db-be35-4c72-a536-fdcbb3e51b1c}

FXData:

</EFOFEX>

1. Let’s examine the perimeters of shapes 1 and 2.

<EFOFEX>
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FXData:

</EFOFEX>

This leads to the interesting result that the procedure does not change the perimeter! It neither converges or diverges, it just stays the same.

**Notes**

This question is an interesting extension to the sum of a geometric series. It requires students to use their knowledge of geometric series in a new context.